部分空間の次元・基底

1 次の部分空間の次元と基底を求めよ。

(1) $W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| \, 3x + 4y + z = 0 \right\}$

(2) $W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| \begin{array}{l} x - y + 2z = 0 \\ 2x + 3y - z = 0 \end{array} \right\}$

(3) $W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| \begin{array}{l} 2x + y - 1 = 0 \\ x - y + 3z = 0 \\ 4x - y + 5z = 0 \end{array} \right\}$

(4) $W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \middle| \begin{array}{l} x + y + z - w = 0 \\ x - y + z + w = 0 \end{array} \right\}$

(5) $W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \middle| \begin{array}{l} x + y + z - 2w = 0 \\ x - y - z + w = 0 \end{array} \right\}$

(6) $W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \middle| \begin{array}{l} 3x + y + z + 2w = 0 \\ x - 2y + z - 3w = 0 \\ 2x + 4y - z - w = 0 \end{array} \right\}$

(7) $W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \middle| \begin{array}{l} x + 2y - 2z + 2w = 0 \\ 2x + 3y - z - w = 0 \\ x + 4y + z + 2w = 0 \end{array} \right\}$

(8) $W = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \middle| \begin{array}{l} 2x_1 + 3x_2 - x_3 + 2x_4 - x_5 = 0 \\ 3x_1 + x_2 - 2x_3 - x_4 + 3x_5 = 0 \\ 4x_1 - 2x_2 + x_3 + x_4 + x_5 = 0 \end{array} \right\}$

2 次の部分空間の次元と基底を求めよ。

(1) $W = \left\langle \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} \right\rangle + \left\langle \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \right\rangle$

(2) $W = \left\langle \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \right\rangle + \left\langle \begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix} \right\rangle$

(3) $W = \left\langle \begin{pmatrix} 3 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 0 \\ 3 \end{pmatrix} \right\rangle + \left\langle \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$

(4) $W = \left\langle \begin{pmatrix} 1 \\ 5 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 1 \\ 4 \end{pmatrix} \right\rangle + \left\langle \begin{pmatrix} 2 \\ -3 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 1 \\ 2 \end{pmatrix} \right\rangle$

(5) $W = \left\langle \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ -2 \\ 1 \end{pmatrix} \right\rangle + \left\langle \begin{pmatrix} 5 \\ 2 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 4 \end{pmatrix} \right\rangle$

(6) $W = \left\langle \begin{pmatrix} 1 \\ 3 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 3 \\ 3 \end{pmatrix} \right\rangle + \left\langle \begin{pmatrix} 3 \\ 2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -3 \\ 0 \end{pmatrix} \right\rangle$

(7) $W = \left\langle \begin{pmatrix} 2 \\ 1 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ -7 \\ 0 \\ 2 \end{pmatrix} \right\rangle + \left\langle \begin{pmatrix} 1 \\ 3 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ -3 \\ 5 \\ 8 \end{pmatrix} \right\rangle$

(8) $W = \left\langle \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} \right\rangle \cap \left\langle \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \right\rangle$

(9) $W = \left\langle \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \right\rangle \cap \left\langle \begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix} \right\rangle$

(10) $W = \left\langle \begin{pmatrix} 3 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 0 \\ 3 \end{pmatrix} \right\rangle \cap \left\langle \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$

(11) $W = \left\langle \begin{pmatrix} 1 \\ 5 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 1 \\ 4 \end{pmatrix} \right\rangle \cap \left\langle \begin{pmatrix} 2 \\ -3 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 1 \\ 2 \end{pmatrix} \right\rangle$

(12) $W = \left\langle \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ -2 \\ 1 \end{pmatrix} \right\rangle \cap \left\langle \begin{pmatrix} 5 \\ 2 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 4 \end{pmatrix} \right\rangle$

(13) $W = \left\langle \begin{pmatrix} 1 \\ 3 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 3 \\ 3 \end{pmatrix} \right\rangle \cap \left\langle \begin{pmatrix} 3 \\ 2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -3 \\ 0 \end{pmatrix} \right\rangle$

(14) $W = \left\langle \begin{pmatrix} 2 \\ 1 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ -7 \\ 0 \\ 2 \end{pmatrix} \right\rangle \cap \left\langle \begin{pmatrix} 1 \\ 3 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ -3 \\ 5 \\ 8 \end{pmatrix} \right\rangle$