1 変数の微分　解答

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(1) $\left(\dfrac{3x+4}{x^2-x+1}\right)' = \dfrac{3(x^2-2+1)-(3x+4)(2x-1)}{(x^2-x+1)^2} = \dfrac{-3x^2-8x+7}{(x^2-x+1)^2}$

(2) $\left(\sqrt{x^3-1}\right)' = \dfrac{3x^2}{2\sqrt{x^3-1}}$

(3) $\left(\dfrac{\log x}{\sin x}\right)' = \dfrac{\frac{1}{x}\sin x - \log x \cdot \cos x}{\sin^2 x} = \dfrac{\sin x - x\log x\cos x}{x\sin^2 x}$

(4) $\left(e^{x\log x}\right)' = (\log x+1)e^{x\log x}$

(5) $\left((\log x)^4\right)' = 4(\log x)^3 \cdot \dfrac{1}{x} = \dfrac{4(\log x)^3}{x}$

(6) $(\log(\cos x))' = \dfrac{-\sin x}{\cos x} = -\tan x$

(7) $(\tan(2x))' = \dfrac{2}{\cos^2(2x)}$

(8) $\left(\dfrac{\sqrt{2x-3}}{x^2+1}\right)' = \dfrac{\frac{1}{2}\frac{2}{\sqrt{2x-3}}(x^2+1)-\sqrt{2x-3}\cdot 2x}{(x^2+1)^2} = \dfrac{-3x^2+6x+1}{\sqrt{2x-3}(x^2+1)^2}$

(9) $\left(\sqrt[3]{x^3-1}\right)' = \dfrac{1}{3}(x^3-1)^{-\frac{2}{3}}\cdot 3x^2 = \dfrac{x^2}{(\sqrt[3]{x^3-1})^2}$

(10) $\left(\sqrt{\tan x}\right)' = \dfrac{1}{2}\dfrac{(\tan x)'}{\sqrt{\tan x}} = \dfrac{1}{2\cos^2 x\sqrt{\tan x}}$

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(1) $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$

(2) $(\cos^{-1} x)' = -\dfrac{1}{\sqrt{1-x^2}}$

(3) $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$

(4) $(\sin^{-1}(x-1))' = \dfrac{1}{\sqrt{1-(x-1)^2}} = \dfrac{1}{\sqrt{x(2-x)}}$

(5) $\left(e^{\sin^{-1} x}\right)' = \dfrac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}$

(6) $$\left(\tan^{-1}\frac{x-1}{x+1}\right)' = \frac{\frac{(x+1)-(x-1)}{(x+1)^2}}{1+\left(\frac{x-1}{x+1}\right)^2} = \frac{2}{(x+1)^2+(x-1)^2} = \frac{1}{1+x^2}$$

(7) $$\left(\cos^{-1} e^x\right)' = -\frac{e^x}{\sqrt{1-e^{2x}}}$$

(8) $$\left(\log(\cos^{-1} x)\right)' = -\frac{1}{\sqrt{1-x^2}\cos^{-1} x}$$

(9) $$\left(x\sin^{-1} x\right)' = \sin^{-1} x + \frac{x}{\sqrt{1-x^2}}$$

(10) $$\left(\frac{\sin^{-1} x}{\cos^{-1} x}\right)' = \frac{\frac{1}{\sqrt{1-x^2}}\cos^{-1} x - \sin^{-1} x\frac{-1}{\sqrt{1-x^2}}}{(\cos^{-1} x)^2} = \frac{\cos^{-1} x + \sin^{-1} x}{\sqrt{1-x^2}(\cos^{-1} x)^2} = \frac{\pi}{2\sqrt{1-x^2}(\cos^{-1} x)^2}$$